Numerical simulation of a singularly perturbed heat equation using the galerkin finite element method

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Parabolic Partial differential equations play a crucial role in a large and varied range of disciplines in science and engineering. It is difficult to solve PDEs analytically; therefore a suitable numerical method is needed to solve them, such as the finite element method (FEM). In this research a linear singularly perturbed heat equation with an external heat source is solved numerically using the finite element method. Discretization of the spatial derivative is done using Galerkin Finite element discretization giving a semi-discrete system of Ordinary differential equations (ODEs). The resulting system of ODEs is integrated numerically using a Backward Euler schemes to obtain the numerical solution. Finally, some numerical simulations are carried out at different perturbation values to show how the solutions behave and also the effect of external heat source is analysed quantitatively. The results of the numerical solution of the singularly perturbed heat equation show that as the perturbation parameter ε decreases, a boundary layer forms at one end of the rod. Also the results show that after introduction of external heat source to the system, the heat source cause the temperature of the system to be higher than before addition of external heat source.