



A noise-suppressing and edge-preserving multiframe super-resolution image reconstruction method

Baraka Jacob Maiseli ^{a,*}, Nassor Ally ^a, Huijun Gao ^b

^a University of Dar es Salaam, College of Information & Communication Technologies, Department of Electronics & Telecommunication Engineering, P. O. Box 35194, Dar es Salaam, Tanzania

^b Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, PR China



ARTICLE INFO

Article history:

Received 25 November 2014

Received in revised form

1 March 2015

Accepted 2 March 2015

Available online 12 March 2015

Keywords:

Convex optimization

Super-resolution

Backward diffusion

Regularization

ABSTRACT

Super-resolution technology is an approach that helps to restore high quality images and videos from degraded ones. The method stems from an ill-posed minimization problem, which is usually solved using the L^2 norm and some regularization techniques. Most of the classical regularizing functionals are based on the Total Variation and the Perona–Malik frameworks, which suffer from undesirable artifacts (blocking and staircasing). To address these problems, we have developed a super-resolution framework that integrates an adaptive diffusion-based regularizer. The model is feature-dependent: linear isotropic in flat regions, a condition that regularizes an image uniformly and removes noise more effectively; and nonlinear anisotropic near boundaries, which helps to preserve important image features, such as edges and contours. Additionally, the new regularizing kernel incorporates a shape-defining parameter that can be automatically updated to ensure convexity and stability of the corresponding energy functional. Comparisons with other methods show that our method is superior and, more importantly, can achieve higher reconstruction factors.

© 2015 Elsevier B.V. All rights reserved.

1. Introduction

Super-resolution image reconstruction is probably the most cost-effective technology to restore high quality images without effecting the circuitry of the image-capturing devices; it reverses the image degradation process. The methods to address this problem can either be multiframe—as in this work—or single-frame based.

The multiframe super-resolution problem is well established [1–10, and references therein] and many researchers have tried to solve it. But, how to enhance image qualities while suppressing noise and preserving important image features remains an open-ended question. The framework these methods rely upon consists of three basic steps:

registration, alignment, and reconstruction, which interact as follows: first, sequence of aliased and noisy low-resolution images are captured and the motions (rotations and translations) between pairs of frames are computed (registration); next, the framework uses the motion values to align the frames onto a high-resolution grid (alignment); and, finally, a high quality image is generated (reconstruction).

Pham et al. proposed an image fusion (super-resolution) method from irregularly sampled data [11]. Their approach is based on the adaptive normalized convolution framework, where a local signal is estimated through a projection onto a subspace. The method is robust and generates sharper images. However, for real video sequences, we noted from the experimental findings that Pham's method reveals undesirable visible artifacts.

In [12], the authors proposed the super-resolution method based on the Total Variational (TV) framework and the Bregman algorithm; and a similar work in [13] uses a spatially

* Corresponding author.

E-mail address: barakamaiseli@yahoo.com (B. Jacob Maiseli).

varying TV model. Despite their ability to preserve useful image details, TV-based approaches are susceptible to blocking artifacts [14,15].

Recently, Knoll et al. proposed a regularizing functional based on the Total Generalized Variation (TGV) framework for Magnetic Resonance Induction (MRI) image reconstruction and denoising [16]. Their method overcomes blocking artifacts inherent in the conventional TV.

For years, nonlinear diffusion-based regularizers have demonstrated good performance in image denoising applications [14,15,17–19, and references therein]; they can simultaneously remove noise and protect important image features. A popular and classical nonlinear diffusion method was proposed by Perona and Malik in 1990 [20]. The Perona–Malik (PM) regularizer generates compelling results, but its corresponding potential function is not entirely convex and may, therefore, evaluate many solutions [21–23]. Other nonlinear diffusion filters were proposed by Charbonnier [24] and Weickert [19,25]. But, we found from experiments that these methods degenerate solutions under intense noise conditions.

Few studies have attempted to combine diffusion-based regularizers and the classical super-resolution model [26–28]. In this work, we propose a new model that combines the two frameworks and addresses the mentioned challenges in the previous methods. We hypothesize that the feature-dependent property of the proposed regularizer will efficiently suppress noise and preserve crucial image details, while avoiding blocking and staircasing effects.

2. Methods

2.1. Nonlinear diffusion processes

Diffusion refers to a physical (transport) process that equilibrates concentration differences while conserving mass [29]. This phenomenon is easily cast by the *Fick's first law*:

$$j = -\phi \cdot \nabla u, \quad (1)$$

which explains the equilibration property that the concentration gradient, ∇u , causes the flux, j , which aims to

compensate it at a rate controlled by the diffusivity, ϕ . In our case, the concentration, u , is the intensity of an image at a particular location.

The fact that diffusion neither creates nor destroys mass is explained by the *continuity equation*:

$$\frac{\partial u}{\partial t} = -\operatorname{div} j. \quad (2)$$

Substituting (1) into (2), we get

$$\frac{\partial u}{\partial t} = \operatorname{div}(\phi \cdot \nabla u). \quad (3)$$

Linear isotropic diffusion occurs when ϕ is constant over the entire image domain. This type of diffusion is unguided and tends to destroy useful image details. Furthermore, if ϕ depends on u , as in

$$\frac{\partial u}{\partial t} = \operatorname{div}(\phi(|\nabla u|) \cdot \nabla u), \quad (4)$$

then diffusion is nonlinear anisotropic. The feature-dependent variable, $\phi(s)$, should be carefully designed to ensure that diffusion is automatic, locally adaptive—linear isotropic in intra regions, and nonlinear anisotropic near edges—and can preserve critical image features (Table 1). In this work, we propose ϕ that achieves these properties.

2.2. Proposed diffusion equation

Our diffusion model, which contains the edge-steering diffusivity, is

$$\frac{\partial u}{\partial t} = \operatorname{div} \left(\frac{2 + \frac{|\nabla u|}{\beta}}{1 + \left(\frac{|\nabla u|}{\beta}\right)^2} \nabla u \right) + \lambda(u - f), \quad (5)$$

where λ , weighing (regularization) parameter; β , shape-defining tuning constant; and $f = u(x, y; t = 0)$, the initial image (u at $t = 0$).

In homogeneous regions, where $|\nabla u| \rightarrow 0$, (5) becomes

$$\frac{\partial u}{\partial t} = C \Delta u + \lambda(u - f), \quad (6)$$

Table 1
Diffusivities and corresponding potentials.

Name of the function	$\phi(\nabla u)$	$\rho(\nabla u), s = \nabla u $
Tikhonov [30]	Constant (identity matrix)	s^2
Total Variation (TV) [31]	$\frac{1}{ \nabla u }$	s
Perona–Malik (PM) [20]	$\frac{1}{1 + \left(\frac{ \nabla u }{K}\right)^2}$	$\frac{K^2}{2} \log \left(1 + \left(\frac{s}{K}\right)^2\right)$
Charbonnier et al. [24]	$\frac{1}{\sqrt{1 + \left(\frac{ \nabla u }{K}\right)^2}}$	$\sqrt{K^4 + K^2 s^2} - K^2$
Ganan and McClure [32]	$\frac{1}{(1 + \nabla u ^2)^2}$	$\frac{s^2}{1 + s^2}$

($C=2$) which is an isotropic linear diffusion (heat equation) that suppresses noise uniformly over the whole image domain. Also, in regions with critical features, where $|\nabla u| \rightarrow \infty$, the diffusivity becomes zero and diffusion stops; thus protecting edges and contours.

Furthermore, expanding the divergence part of (5) yields

$$\begin{aligned} \frac{\partial u}{\partial t} = & C \operatorname{div} \left(\frac{\nabla u}{1 + \left(\frac{|\nabla u|}{\beta} \right)^2} \right) + \beta \operatorname{div} \left(\frac{\nabla u}{|\nabla u|} \right) \\ & + \beta \operatorname{div} \left(\frac{-1}{|\nabla u| \left(1 + \left(\frac{|\nabla u|}{\beta} \right)^2 \right)} \nabla u \right) + \lambda(u - f), \end{aligned} \quad (7)$$

where the terms (left to right) are defined as follows: first, the Perona–Malik (PM) [20]; second, the nonlinear Total Variation (TV) [31]; third, the backward diffusion model that may sharpen edges and contours [21]; and fourth, the fidelity term. The combined effect of these terms (PM, TV, and backward diffusion), therefore, is expected to provide effective smoothing, preserve important details, and sharpen edges.

From the new evolution equation (5), as $t \rightarrow \infty$ the evolving solution, u , approaches a stationary function that minimizes the energy functional (ignore the standard terms, data and fidelity, for simplicity)

$$\rho(|\nabla u|) = \int_{\Omega} \frac{\frac{|\nabla u|}{\beta} \left(2 + \frac{|\nabla u|}{\beta} \right)}{1 + \left(\frac{|\nabla u|}{\beta} \right)^2} d\Omega. \quad (8)$$

Solving (8) using the Euler–Lagrange principle gives the new potential

$$\rho(|\nabla u|) = \beta^2 \log(\beta^2 + |\nabla u|^2) + \beta |\nabla u| - \beta^2 \arctan\left(\frac{|\nabla u|}{\beta}\right) - 2\beta^2 \log(\beta), \quad (9)$$

which possesses some useful properties: one, the first two terms on the right side ($\beta^2 \log(\beta^2 + |\nabla u|^2)$ and $\beta |\nabla u|$) are the potentials similar to those of the Perona–Malik and the TV, respectively (Table 1); and two, it achieves convexity when $\beta > \frac{2}{1+\sqrt{5}} |\nabla u|$ (Fig. 2) (see Theorem 1 and the corresponding proof).

Theorem 1. A functional $\rho(s)$ in (9) is strictly convex if $\rho''(s) > 0 \forall s$.

Proof. From the energy functional in (9),

$$\rho''(s) = \frac{2\beta^2(\beta^2 + \beta s - s^2)}{(\beta^2 + s^2)^2}.$$

Intuitively, we see that $F'' > 0$ —a sufficient condition (but not necessary) for convexity—if $(\beta^2 + \beta s - s^2) > 0$ or $\beta > \frac{2}{1+\sqrt{5}} s$. $\square$

Now, we desire to explain the mechanical properties of (5) that allows it to sharpen edges and preserve useful image details: consider a pixel location, (x, y) , in the image where $|\nabla u(x, y)| \neq 0$, and define two orthogonal vectors: normal, $N(x) = (u_x, u_y)/|\nabla u|$ and tangential, $T(x) = (-u_y, u_x)/|\nabla u|$,

where u_x and u_y are the first order partial derivatives and $\langle N, T \rangle = 0$ is a scalar inner product. If u_{TT} and u_{NN} are respectively the second order partial derivatives in the directions of T and N , then the regularization part of (5) can compactly be written in terms of these components as

$$R(|\nabla u|) = \left[\frac{2 + \frac{|\nabla u|}{\beta}}{1 + \left(\frac{|\nabla u|}{\beta} \right)^2} \right] u_{TT} + \left[\frac{2 \left(1 + \frac{|\nabla u|}{\beta} - \left(\frac{|\nabla u|}{\beta} \right)^2 \right)}{\left(1 + \left(\frac{|\nabla u|}{\beta} \right)^2 \right)^2} \right] u_{NN} \quad (10)$$

where

$$u_{TT} = \frac{1}{|\nabla u|^2} (u_x^2 u_{yy} + u_y^2 u_{xx} - 2u_x u_y u_{xy})$$

and

$$u_{NN} = \frac{1}{|\nabla u|^2} (u_x^2 u_{xx} + u_y^2 u_{yy} + 2u_x u_y u_{xy}).$$

The goal of the u_{TT} component is to guide the regularization process along the contours of an image; the u_{NN} component acts perpendicular to the contours. Sharper and more enhanced edges are produced if these components are properly balanced. We needed to dominate u_{TT} in regions with critical features (edges and contours), and to allow co-existence of both u_{TT} and u_{NN} in flat regions. The proposed model satisfies these requirements: near edges ($|\nabla u| \rightarrow \infty$), u_{NN} diminishes faster than u_{TT} because the denominator of its coefficient is larger than that of the u_{TT} component. This dominates the u_{TT} component, which is necessary to preserve edges. Furthermore, in flat regions ($|\nabla u| \rightarrow 0$) Eq. (10) reduces to

$$R(|\nabla u|) = C(u_{TT} + u_{NN}) = C\Delta u, \quad (11)$$

which behaves like the normal heat equation that performs uniform regularization. Also, Eq. (10) shows that the new model introduces backward diffusion when

$$2 \left(1 + \frac{|\nabla u|}{\beta} - \left(\frac{|\nabla u|}{\beta} \right)^2 \right) < 0 \quad \text{or} \quad \frac{|\nabla u|}{\beta} > \frac{1 + \sqrt{5}}{2}; \quad (12)$$

a condition that may enhance edges [21], and that is suitable for the super-resolution and other image enhancement applications.

In the next subsection, we describe the super-resolution problem and explain how its ill-posedness can be addressed using the proposed regularizing functional.

2.3. Multiframe super-resolution model

Optical imaging systems tend to introduce undesirable artifacts when capturing and processing input images. The common artifacts, usually caused by hardware imperfections, are warping, blurring, decimation, and noise. To model the degradation process, consider that the unknown ideal image, u , is acted upon by the operators W_k (warping), B_k (blurring), D_k (decimation), and e_k (additive noise), where k is the frame number (Fig. 1). The degradation operators: W_k , rotates and translates u ; B_k , blurs u and destroys its focus; and D_k , reduces the size of u . The degradation model applies these

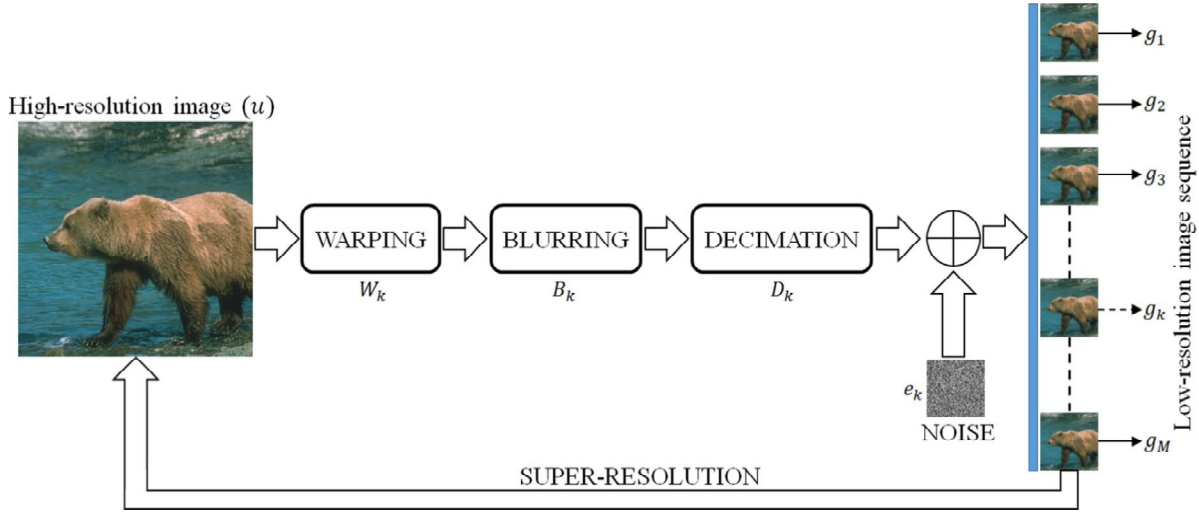


Fig. 1. Degradation model for the multiframe super-resolution.

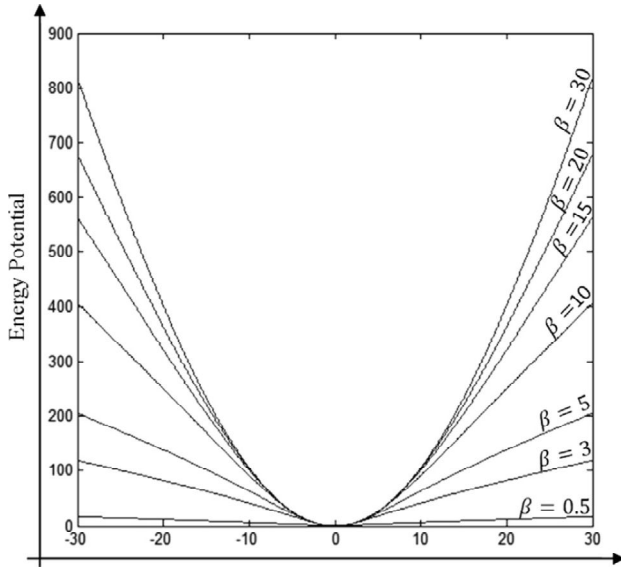


Fig. 2. Proposed energy functional at different values of the shape-defining parameter, β . (The condition for convexity is $\beta > \frac{2}{1+\sqrt{5}}|\nabla u|$.)

operators onto u to generate a sequence of M low-resolution images: $g_1, g_2, \dots, g_k, \dots, g_M$.

Following the degradation model, the multiframe super-resolution problem is modeled as

$$g_k = W_k B_k D_k u + e_k. \quad (13)$$

The objective is to minimize the amount of noise (e_k) in u using the L^2 norm. Therefore, the minimization problem becomes

$$\min_u \left\{ \frac{1}{2M} \sum_{k=1}^M \|g_k - W_k B_k D_k u\|_{L^2(\Omega)}^2 \right\}, \quad (14)$$

where Ω is the image support.

The minimization equation (14) is ill-posed—and thus prone to instability and multiple solutions—because it contains insufficient number of observed low-resolution images

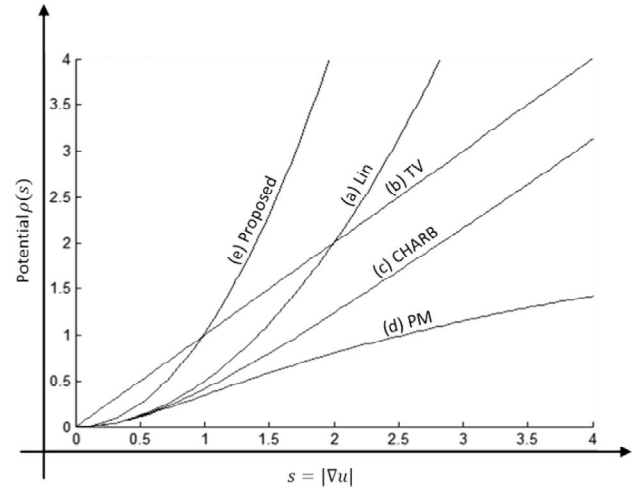


Fig. 3. Potentials of some methods: (a) linear forward diffusion (Lin), $\rho(s) = \frac{1}{2}s^2$ [41]; (b) TV, $\rho(s) = s$ [31]; (c) Charbonnier (CHARB), $\rho(s) = \sqrt{K^4 + K^2 s^2} - K^2$, $K = 1$ [24]; (d) Perona–Malik (PM), $\rho(s) = (K/2) \log(1 + (s/K)^2)$, $K = 1$ [20]; and (e) our potential, $\beta = 1$.

(M). One way to address this ill-posedness is to incorporate a regularizing potential into the formulation [33–37].

In practice, regularizing potentials grow monotonically, and can either be convex [15,24,38]—thus well-posed and ability to guide the evolving solution to a global minimum—or non-convex [20] that are susceptible to multiple solutions [39], as proved by Höllig [40]. Fig. 3 compares the convexities of various potentials (CHARB, PM, Lin, TV, and ours); the corresponding diffusivities are shown in Fig. 4, with our diffusivity demonstrating the nonlinearity behavior similar to TV, CHARB, and PM. Note that if the tuning constant, β , in the proposed model explodes to infinity the diffusion coefficient approaches a constant quantity, thus making the smoothing functional linear isotropic (curve (e) in Fig. 4 becomes constant).

Now, combining the new regularizing potential, $\rho(|\nabla u|)$, in (9), the fidelity potential term, and the minimization problem from the classical multiframe super-resolution

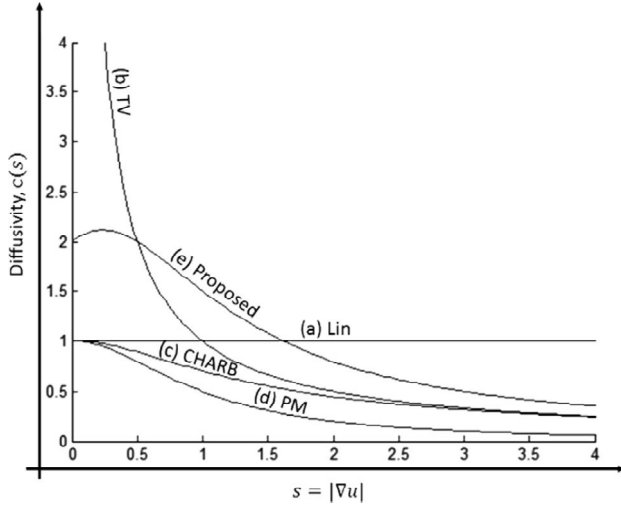


Fig. 4. Diffusivities of some methods: (a) linear forward diffusion (Lin), $c(s)=1$ [41]; (b) TV, $c(s)=1/s$ [31]; (c) Charbonnier (CHARB), $c(s)=1/\sqrt{1+(s/K)^2}$, $K=1$ [24]; (d) Perona–Malik (PM), $c(s)=1/(1+(s/K)^2)$, $K=1$ [20]; and (e) our diffusivity, $\beta=1$.

model in (14), we get

$$\min_u \left\{ \frac{1}{2M} \sum_{k=1}^M \|g_k - W_k B_k D_k u\|_{L^2(\Omega)}^2 + \|\rho(|\nabla u|)\|_{L^1(\Omega)} + \frac{1}{2} \lambda \|u - f\|_{L^2(\Omega)}^2 \right\}. \quad (15)$$

The corresponding dynamical system of the Euler–Lagrange form of (15) is

$$\frac{\partial u}{\partial t} = \frac{1}{M} \sum_{k=1}^M W_k^T B_k^T D_k^T (W_k B_k D_k u - g_k) + \operatorname{div} \left(\frac{2 + \frac{|\nabla u|}{\beta}}{1 + \left(\frac{|\nabla u|}{\beta} \right)^2} \nabla u \right) + \lambda(u - f), \quad (16)$$

where the terms on the right side (in that order) are defined as follows: first, the super-resolution model; second, the regularizing functional; and third, the fidelity.

2.4. Numerical implementation details

We used the explicit numerical scheme (Fig. 5) to implement the new regularizing functional in (5) or middle term in (16). This scheme was selected for its simplicity, ability to generate more accurate results, and stability when the Courant–Friedrichs–Lewy condition is obeyed ($0 < d \leq 0.25$, where d is the iteration step) [42]. Therefore, the divergence, $\operatorname{div}(\cdot)$, part of the regularizing functional is discretized as follows:

$$\operatorname{div}_{i,j}^{(n)} = C_{Ni,j}^{(n)} \nabla_N u_{i,j} + C_{Si,j}^{(n)} \nabla_S u_{i,j} + C_{Ei,j}^{(n)} \nabla_E u_{i,j} + C_{Wi,j}^{(n)} \nabla_W u_{i,j}, \quad (17)$$

where

$$C_{Ni,j}^{(n)} = \frac{2 + \frac{|\nabla_N u_{i,j}|}{\beta}}{1 + \left(\frac{|\nabla_N u_{i,j}|}{\beta} \right)^2}, \quad C_{Si,j}^{(n)} = \frac{2 + \frac{|\nabla_S u_{i,j}|}{\beta}}{1 + \left(\frac{|\nabla_S u_{i,j}|}{\beta} \right)^2}$$

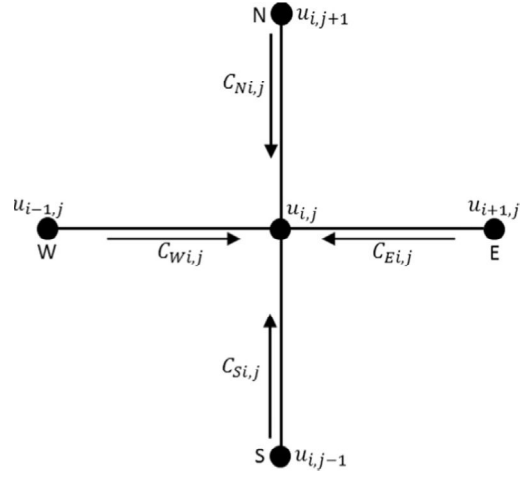


Fig. 5. Explicit numerical scheme: N, North; S, South; E, East; and W, West. The diffusion conduction coefficients in the N, S, E, and W directions are, respectively, $C_{Ni,j}$, $C_{Si,j}$, $C_{Ei,j}$, and $C_{Wi,j}$.

$$C_{Ei,j}^{(n)} = \frac{2 + \frac{|\nabla_E u_{i,j}|}{\beta}}{1 + \left(\frac{|\nabla_E u_{i,j}|}{\beta} \right)^2} \quad \text{and} \quad C_{Wi,j}^{(n)} = \frac{2 + \frac{|\nabla_W u_{i,j}|}{\beta}}{1 + \left(\frac{|\nabla_W u_{i,j}|}{\beta} \right)^2}$$

are the diffusion coefficients and $\nabla_N u_{i,j} = u_{i,j+1} - u_{i,j}$, $\nabla_S u_{i,j} = u_{i,j-1} - u_{i,j}$, $\nabla_E u_{i,j} = u_{i+1,j} - u_{i,j}$, and $\nabla_W u_{i,j} = u_{i-1,j} - u_{i,j}$ are the gradients of $u_{i,j}$, both in the North, South, East, and West directions, respectively. Furthermore, the steepest-descent numerical implementation of the regularized super-resolution model in (16) is

$$u_{i,j}^{(n+1)} = u_{i,j}^{(n)} - d(H_{i,j}^{(n)} - M\tau \operatorname{div}_{i,j}^{(n)} + \lambda M(u_{i,j} - f_{i,j})), \quad (18)$$

where τ is a tuning constant that determines the rate and stability of convergence ($\tau = 0.05$ was ideal for our case), $u_{i,j}^{(0)}$ is the initial estimation of the high-resolution image ($f_{i,j} = u_{i,j}^{(0)}$, and is obtained using the nearest neighbor interpolation), and

$$H_{i,j}^{(n)} = \frac{1}{M} \sum_{k=1}^M W_k^T B_k^T D_k^T (W_k B_k D_k u_{i,j}^{(n)} - (g_k)_{i,j}^{(n)})$$

is a super-resolution result after n iterations. In the experiments, we used the L^2 norm as an iteration-stopping mechanism; and the value of n (number of iterations) ranged between 16 and 30 when $d = 0.20$, $M = 10$, $\tau = 0.05$, $\lambda = 0.05$, and resolution factor equals four.

The full implementation codes for the proposed model are included in the MATLAB central database.¹

2.5. Quality assessment of the super-resolution results

Two quality metrics were used to quantify the performance of the super-resolution results from different methods, namely Peak-Signal-to-Noise-Ratio (PSNR) [43] and Structural Similarity (SSIM) [44]. The PSNR measures the strength of a

¹ <http://www.mathworks.com/matlabcentral/fileexchange/49819-superresolution-application>

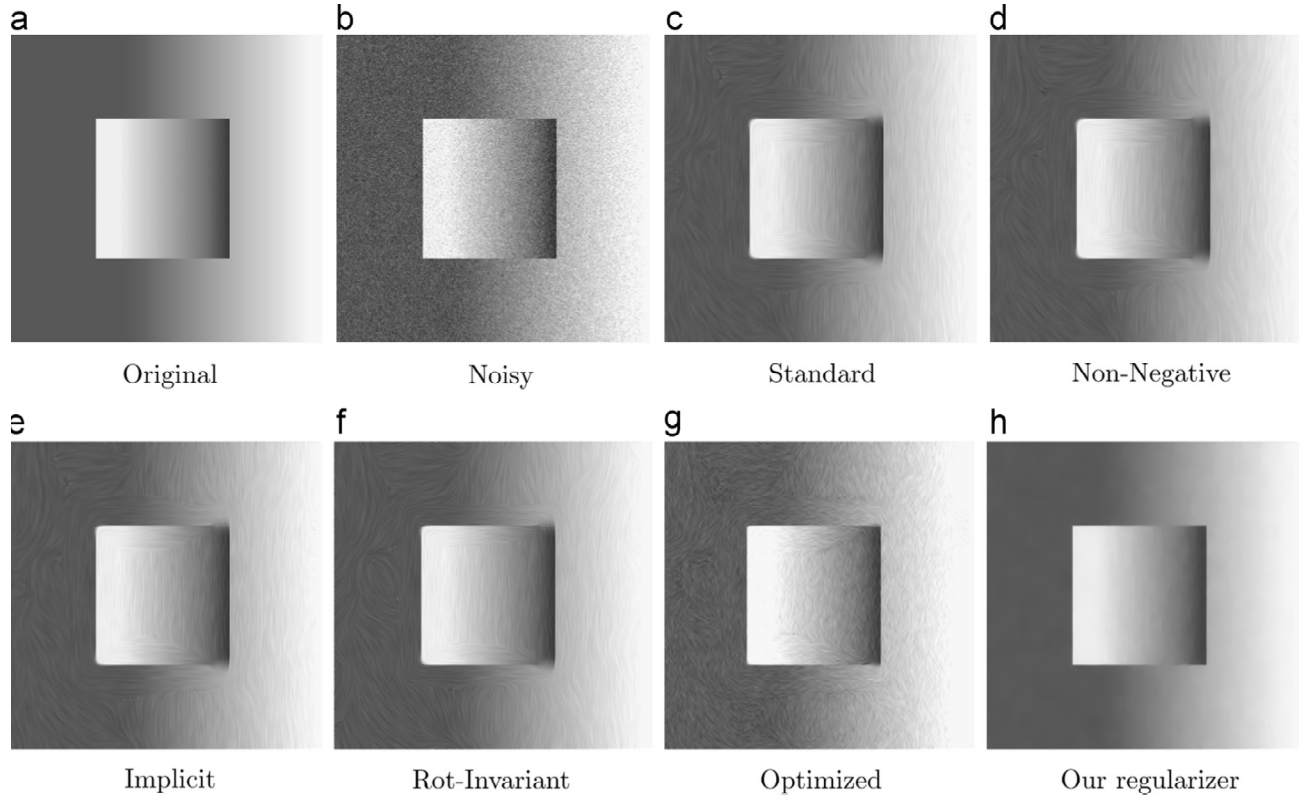


Fig. 6. Coherence-enhancing diffusion regularizers (different schemes) versus our regularizer. Noise statistics: random, mean (μ)=0, standard deviation (σ)=15. (a) Original. (b) Noisy. (c) Standard. (d) Non-negative. (e) Implicit. (f) Rot-Invariant. (g) Optimized. (h) Our regularizer.

signal with respect to noise, and is given by

$$\text{PSNR}(u, f) = 10 \log \left(\frac{255^2 PQ}{\|u - f\|_2^2} \right), \quad (19)$$

where u is the super-resolved image, f is the original image, P and Q are, respectively, the total number of rows and columns in either u or f . Larger value of PSNR signifies higher performance, and vice versa.

Though popular and intuitive metric to understand, researchers have reported the inconsistency of the PSNR with the human visual system. The quantity only considers signal information in the image, and not structural and statistical details that entails visual appearance—from (19), PSNR simply takes as inputs u and f , which barely hold intensity values, and determines the error between them. Hence, a visually appealing image may have a lower PSNR value.

In [44], Wang et al. proposed a quality metric (SSIM) that considers the perceptual changes in the degraded image with regard to its structural information. The metric reveals inter-dependency among pixels in the image that reflects visual qualities. The value of SSIM is between 0 and 1, and higher value in the range means that the reconstructed image is visually close to the reference image. More formally, SSIM is given by the formula

$$\text{SSIM}(u, f) = \frac{(2\mu_u \mu_f + c_1)(2\sigma_{uf} + c_2)}{(\mu_u^2 + \mu_f^2 + c_1)(\sigma_u^2 + \sigma_f^2 + c_2)}, \quad (20)$$

where μ_u , the average of u ; μ_f , the average of f ; σ_u^2 , the variance of u ; σ_f^2 , the variance of f ; σ_{uf} , the covariance of u and f ; and c_1 and c_2 are the stabilizing constants.

3. Experiments

3.1. Simulations

3.1.1. Experiment 1

The aim of this experiment was to observe the efficacy of the proposed regularizer in (5) with respect to other classical regularizers. The experiment followed the following steps: firstly, we synthesized a piecewise constant image and added to it noise with mean, μ , of zero and with standard deviation, σ , of 15; secondly, we applied a coherence-enhancing diffusion regularizer [19], which promotes a strong-directed anisotropic process, at different schemes—standard, non-negative, implicit, rotation-invariant, and optimized—that differ in their directionality properties [45–47]; thirdly, our regularizing functional was applied to the noisy image (Fig. 6). Next, we doubled the noise level in the input image ($\mu = 0$, $\sigma = 30$) and applied the nonlinear diffusion regularizers—PM [20], Charbonnier et al. [24], Weickert et al. [25], and ours—to generate the results in Fig. 7. Note that noise was doubled in this experiment because diffusion-based regularizers could easily reveal artifacts under severe noise conditions.

Additionally, a synthesized noisy image in Fig. 8(a), adapted from the work of Knoll et al. [16], was denoised using the variational methods (Total Variation, TV and Total

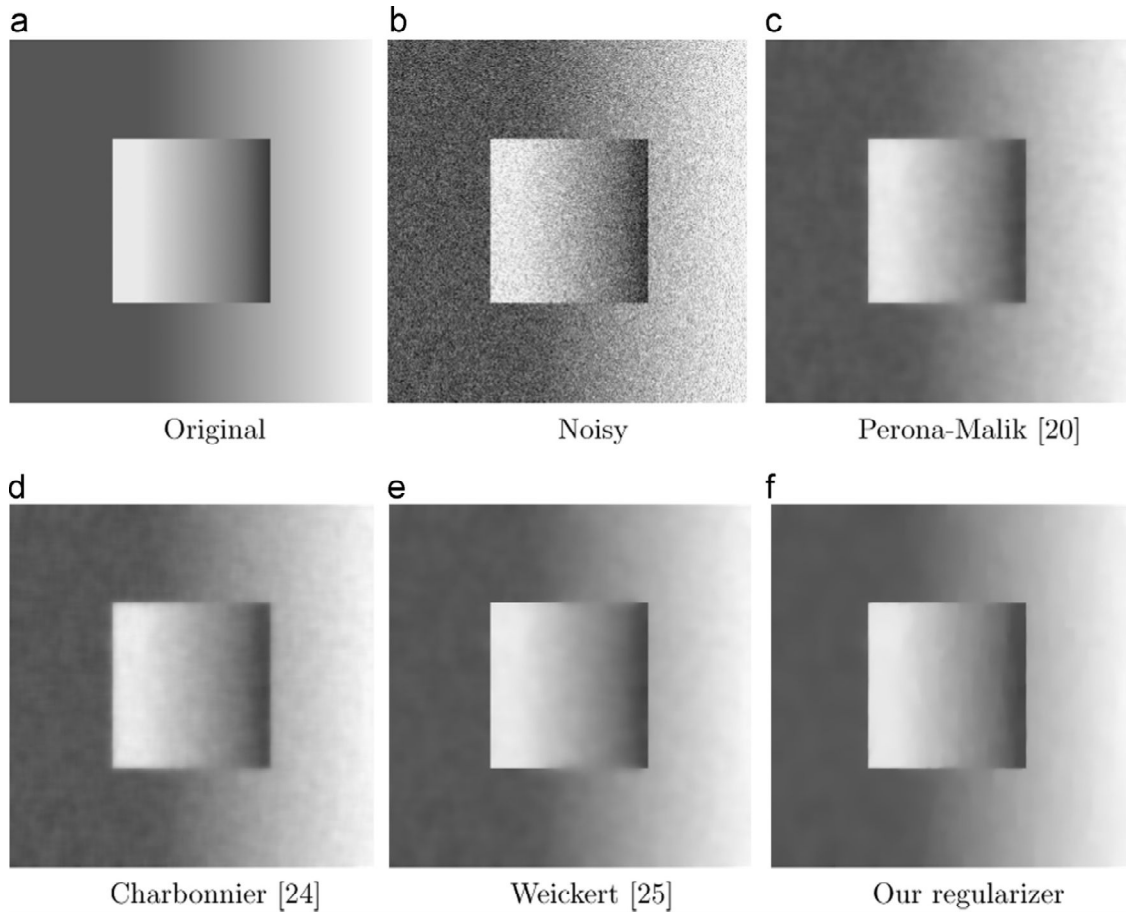


Fig. 7. Diffusion-based regularizers. Noise statistics: random, mean (μ) = 0, standard deviation (σ) = 30. (a) Original. (b) Noisy. (c) Perona–Malik [20]. (d) Charbonnier [24]. (e) Weickert [25]. (f) Our regularizer.

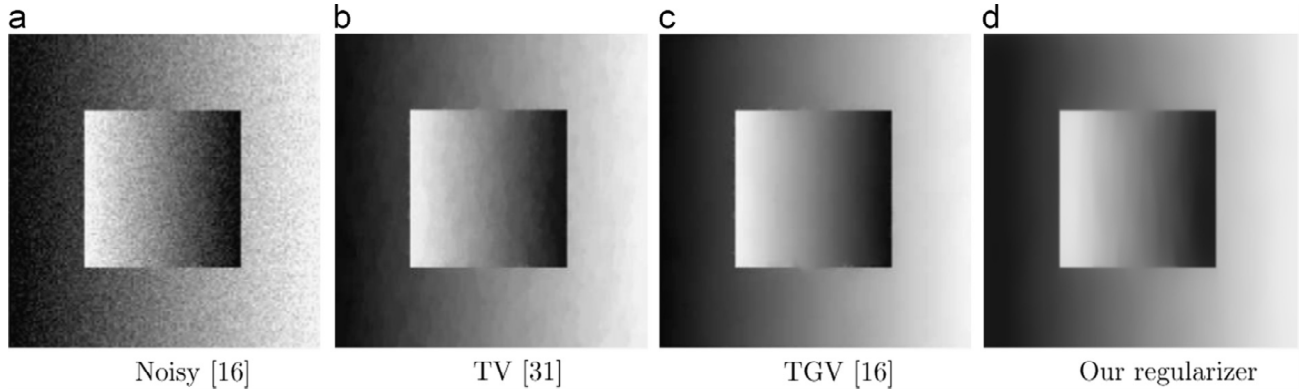


Fig. 8. Variational-based regularizers (Total Variation, TV and Total Generalized Variation, TGV) versus our diffusion-steering smoothing functional. (a) Noisy [16]. (b) TV [31]. (c) TGV [16]. (d) Our regularizer.

Generalized Variation, TGV) and the new regularizer (Fig. 8).

3.1.2. Experiment 2

Standard original images were downloaded from the Berkeley Image Segmentation database [48] and the public image database.² Every image was resized (for presentation purposes in this paper) and degraded: warped, the geometric transformation matrix (W_k in Fig. 1) was

obtained by randomly generating numbers between -1 and 1 ; blurred, Gaussian-shaped blur of 1σ kernel width; decimated, 0.25 sampling rate; and noised, additive random noise (e_k in Fig. 1) was determined by the equation

$$e_k = n \left(\sqrt{\frac{\sum (I_{LR})_k^2}{\sum n^2}} \right) 10^{-\text{SNR}/20} \quad [49] \quad (21)$$

I_{LR} , low-resolution version of u ; $n = \text{randn}(\text{size}(I_{LR}))$, function in MATLAB; and SNR, signal-to-noise ratio (Fig. 10, first row). Consequently, the corresponding sequences of

² http://www.petitcolas.net/watermarking/image_database/

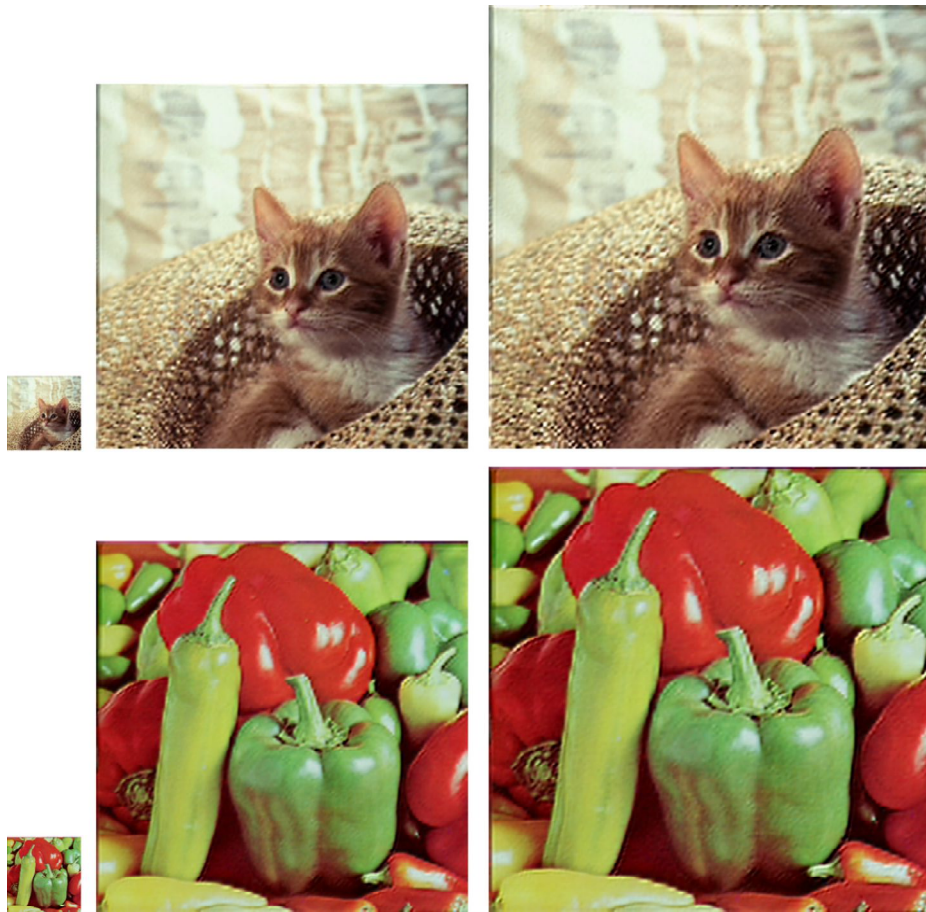


Fig. 9. Our super-resolution method applied at different reconstruction factors. From left to right: low-resolution (128×128 , scaled for display purposes), factors five and six.



Fig. 10. First row, original images (from left to right: Peppers, Cat, Lena, and Baboon) and second row, one of the corresponding low-resolution frame.

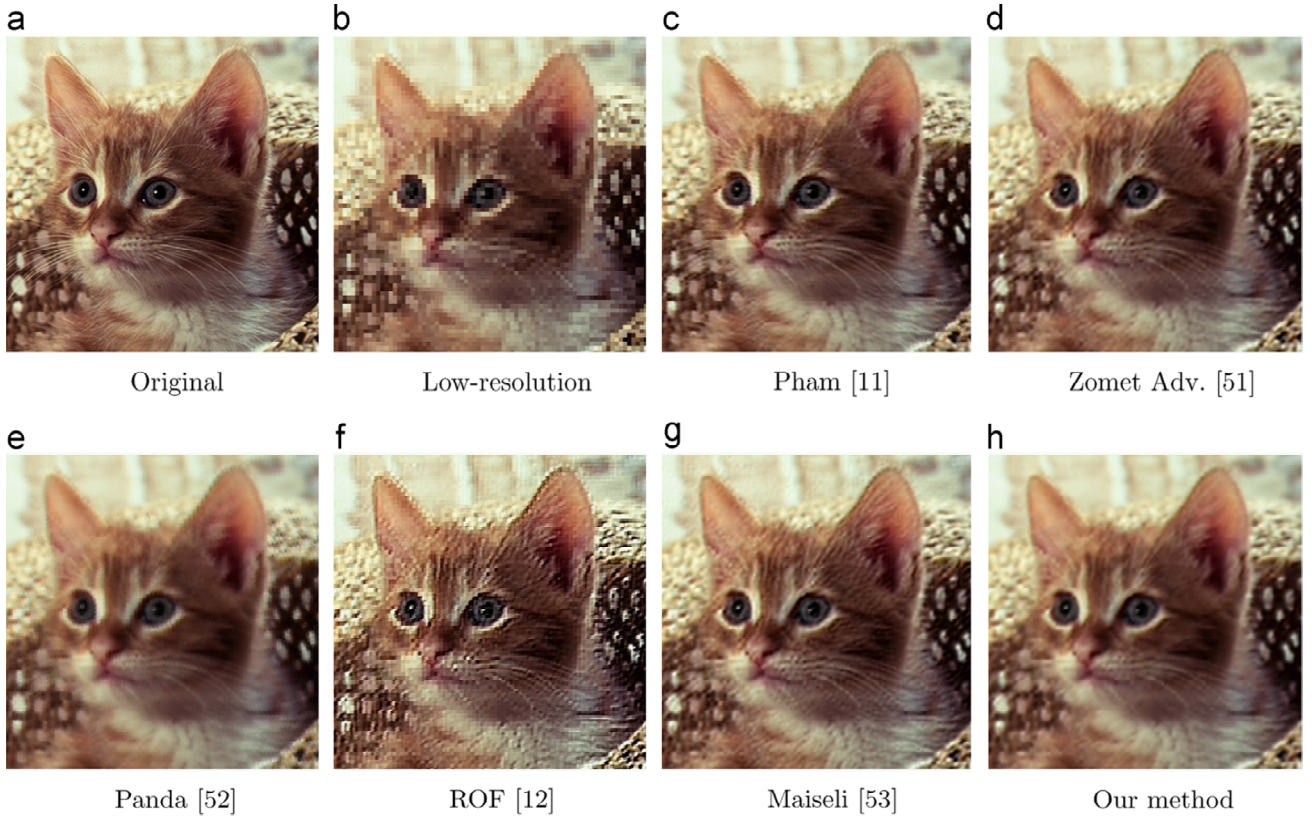


Fig. 11. Super-resolution results of “Cat”. (a) Original. (b) Low-resolution. (c) Pham [11]. (d) Zomet Adv. [51]. (e) Panda [52]. (f) ROF [12]. (g) Maiseli [53]. (h) Our method.

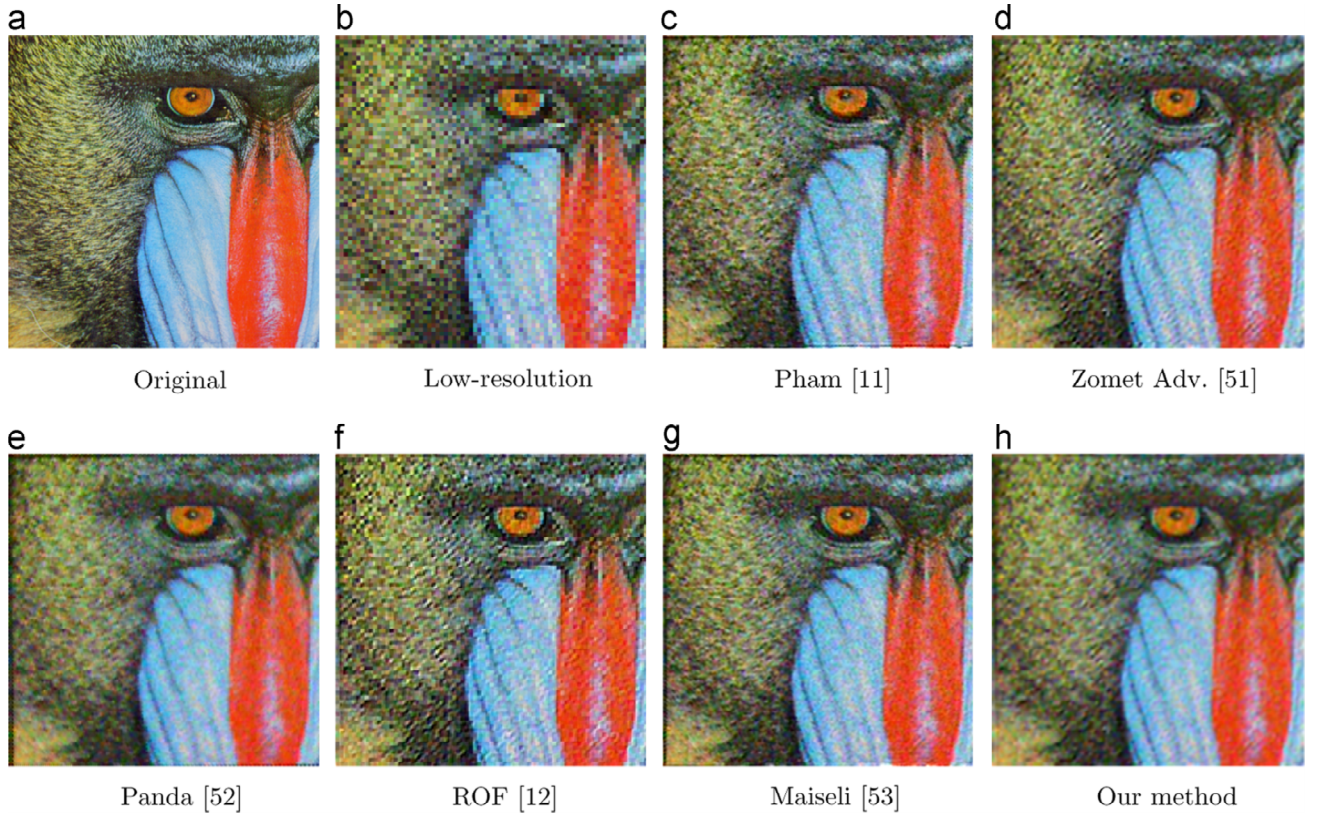


Fig. 12. Super-resolution results of “Baboon”. (a) Original. (b) Low-resolution. (c) Pham [11]. (d) Zomet Adv. [51]. (e) Panda [52]. (f) ROF [12]. (g) Maiseli [53]. (h) Our method.

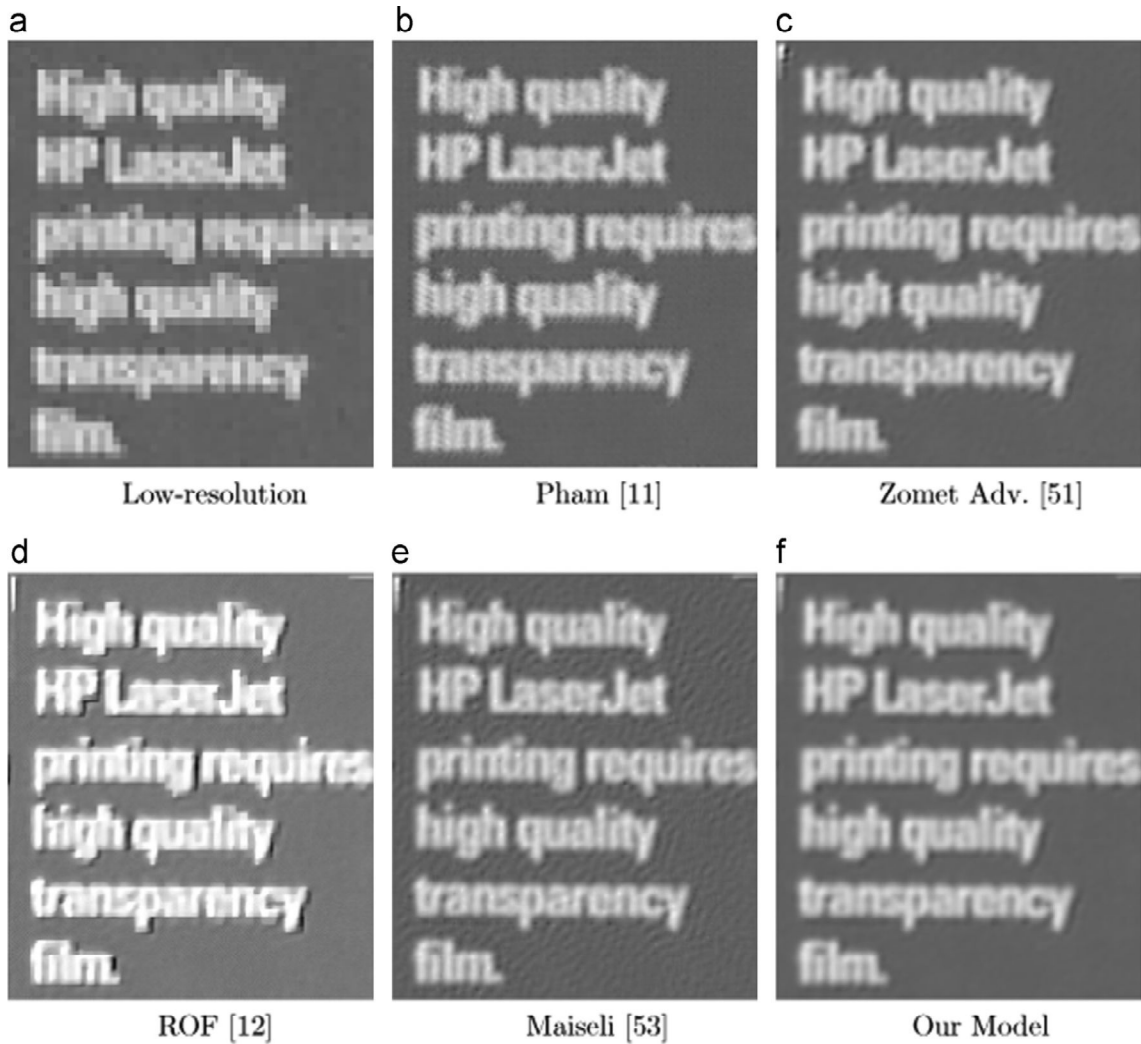


Fig. 13. Super-resolution results from the real video sequences of “Text”. (a) Low-resolution. (b) Pham [11]. (c) Zomet Adv. [51]. (d) ROF [12]. (e) Maiseli [53]. (f) Our model.

ten low-resolution frames were generated from each of the high-resolution image (Fig. 10, second row). Next, we applied our method—along with the Keren registration algorithm [50]—and some other classical methods: Pham [11], Zomet with Charbonnier penalty [51], Panda [52], ROF [12], and Maiseli [53] to generate high-resolution images, each with an upscale factor of four, from the corresponding degraded image sequences (Figs. 11 and 12).

3.1.3. Experiment 3

In this experiment, we tested the robustness and performance of our method at resolution factors above four (Fig. 9). The focus was to visually perceive the response of the proposed approach under higher resolution conditions. Note that most traditional methods are restricted to reconstruction factors between two and four; such methods poorly perform at higher factors.

3.2. Real environment

This experiment used the real video sequences from the *Multidimensional Signal Processing (MDSP) Research*

*Group*³ as inputs to the super-resolution methods: Pham [11], Zomet Adv. [51] (advanced Zomet method with Charbonnier penalty [24]), ROF [12], Maiseli [53], and ours. In particular, ten first frames from each of the two videos —“Text”, 30 grayscale frames, 57×49 resolution; and “Disk”, 26 grayscale frames, 57×49 resolution—with an approximate global motion, as determined using the Keren registration approach [50], were used to reconstruct the corresponding high quality images of $4 \times$ resolution (Figs. 13 and 14).

4. Results and discussions

The visual results from both simulated images and real video sequences show that our method produces some promising results. In particular, the new regularizing functional suppresses noise effectively and preserves semantically crucial features; other methods reveal visible artifacts (Figs. 6–8). Furthermore, the super-resolution results

³ <http://users.soe.ucsc.edu/~milanfar/software/sr-datasets.html>

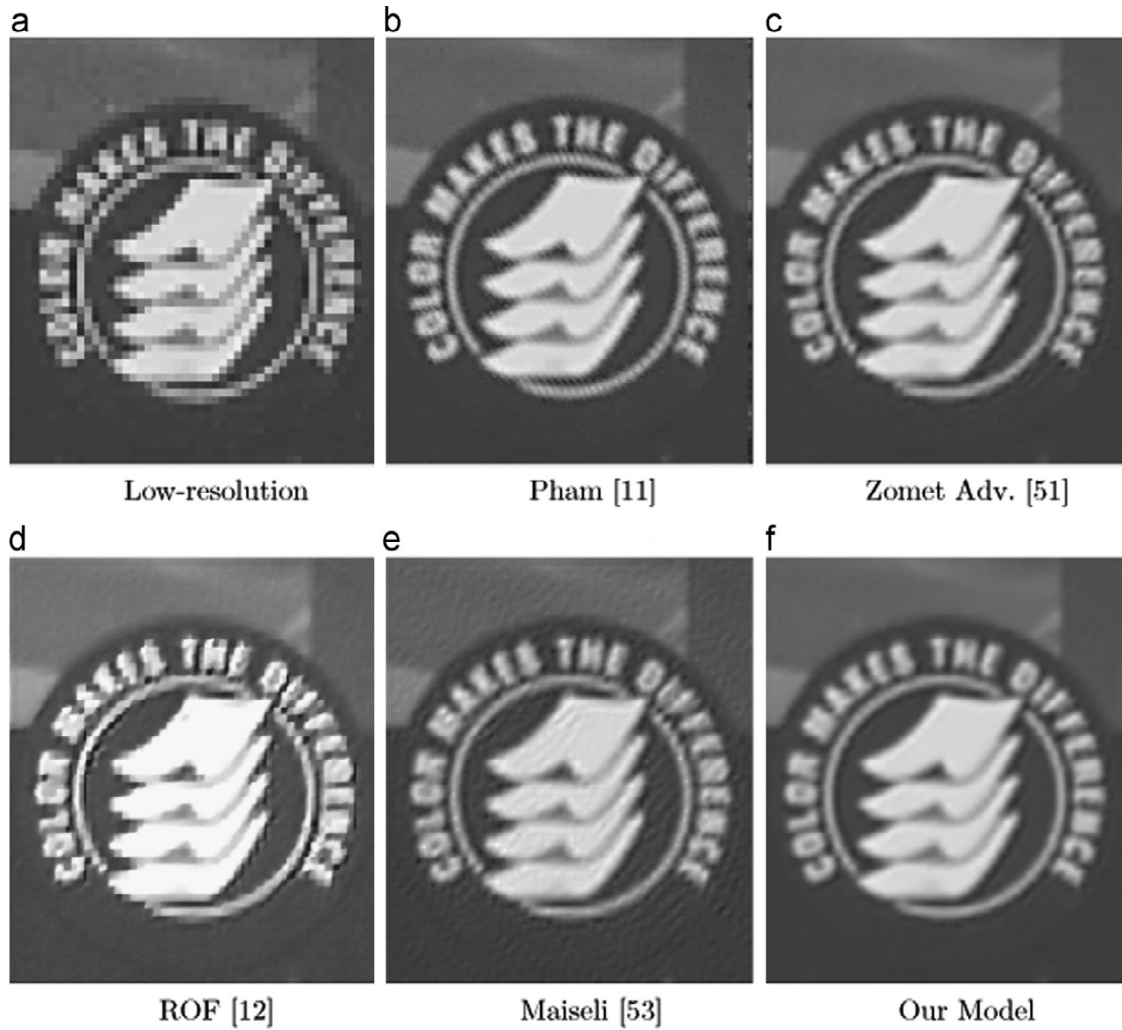


Fig. 14. Super-resolution results from the real video sequences of “Disk”. (a) Low-resolution. (b) Pham [11]. (c) Zomet Adv. [51]. (d) ROF [12]. (e) Maiseli [53]. (f) Our model.

generated by the new method are smoother and contain little artifacts compared with other methods (Figs. 11–14), and this is attributed to the better regularity properties of our potential in Eq. (9). Observations from Figs. 11(h) and 12(h) further suggest that the proposed method tends to slightly blur the results. This may, in part, be caused by the inefficient deblurring function in the super-resolution degradation model (B_k in Fig. 1). In the future, we intend to address this issue by developing a more accurate, efficient, and robust deblurring function and integrate it into the current system.

Also, Tables 2 and 3 show that the new method achieves higher values of PSNR and SSIM compared with most other methods. Note that the Zomet method produces a slightly higher PSNR for a “Cat” image compared with our method (Table 2). Perhaps the classical methods underperform numerically because they mostly suffer from visual artifacts (Figs. 11–14) that degrade their objective qualities. Recall that both PSNR and SSIM are based on the original image; thus, artifacts, noise, or misalignment problems between the restored and original images—resulted from the method failing to properly register the input video sequences—can significantly lower the values of these quality metrics. Consider, for example, the visual results of Pham and ours

(Fig. 11(c) and (h)); although our result looks blurrier than Pham’s, it is smoother and contains fewer artifacts. This reason, together with the more accurate registration algorithm we selected, may have caused the proposed method to produce relatively higher quality values.

Experiment 3 gives an interesting behavior about the new method, that it may achieve higher reconstruction factors—up to six or, perhaps, even more for input images that are heavily aliased. The promising performance of our method at higher factors is caused by the shape-defining parameter, β , in (16) that is automatically updated ($\beta = \epsilon + \frac{2}{1+\sqrt{5}}|Vu|$, where $\epsilon > 0$ is very small) in the program to ensure convexity of the potential in (9), thus promoting stability of the evolving solution over longer periods. Most other multiframe super-resolution methods, however, generate unsatisfactory results at higher resolutions.

5. Conclusion

In this work, we have presented a regularized multiframe super-resolution model that suppresses noise and reconstructs detailed images. Both qualitative and

Table 2

Peak-Signal-to-Noise-Ratio (PSNR) measurements.

Image	PSNR					
	Pham [11]	Zomet Adv. [51]	Panda [52]	ROF [12]	Maiseli [53]	Our model
Peppers	28.12	27.39	27.87	29.20	28.93	30.38
Cat	27.12	28.12	25.33	23.67	27.23	28.01
Lena	27.89	28.10	27.38	29.07	28.99	29.89
Baboon	27.11	26.90	28.26	26.78	30.43	29.79

Table 3

Structural SIMilarity (SSIM) measurements.

Image	SSIM					
	Pham [11]	Zomet Adv. [51]	Panda [52]	ROF [12]	Maiseli [53]	Our model
Peppers	0.8076	0.7819	0.7718	0.7899	0.7910	0.8700
Cat	0.7534	0.7610	0.7392	0.6991	0.7800	0.8695
Lena	0.8417	0.8688	0.8529	0.8832	0.8767	0.9013
Baboon	0.6912	0.6723	0.7002	0.7832	0.8021	0.7908

quantitative results prove that the new model outperforms some other traditional super-resolution methods: protects edges, suppresses noise, enhances image features, and fairly avoids undesirable artifacts. We have explained how our method operates adaptively in response to the local image features, and further demonstrated its behavior through real experiments and simulations.

The subjective results demonstrate that our method slightly blurs the final images. This may partly be caused by lack of the appropriate deblurring operator or perhaps inaccurate estimation of the point-spread-function. The next phase of this research is to develop a more accurate and an effective deblurring component to address the problem. However, objective results show that the new method produces higher PSNR and SSIM values, which signals the importance of the method in machine-related tasks.

We have provided an experiment to prove that the proposed method may generate appealing results at higher magnification factors (up to six, for instance) for severely aliased input images. Our goal in the future is to test this finding on real video sequences. Also, we desire to embed the new algorithm into the actual imaging hardware, such as a digital camera, and measure its performance at higher resolutions.

Acknowledgments

The authors of this paper would like to sincerely thank Prof. Knoll F. from *Center for Advanced Imaging Innovation and Research (CAI²R)* for his technical support on the TGV method for MRI image reconstruction. It is through his guidance that we managed to compare the TGV denoising method with our regularizing functional. We further confirm that none of the organization funded this research.

References

- [1] C. Chen, H. Liang, S. Zhao, Z. Lyu, M. Sarem, A novel multi-image super-resolution reconstruction method using anisotropic fractional order adaptive norm, *Vis. Comput.* (2014) 1–15.
- [2] D.P. Kouri, E.A. Shields, Efficient multiframe super-resolution for imagery with lateral shifts, *Appl. Opt.* 53 (2014) F1–F9.
- [3] B.N. Narayanan, R.C. Hardie, E.J. Balster, Multiframe adaptive Wiener filter super-resolution with jpeg2000-compressed images, *EURASIP J. Adv. Signal Process.* 2014 (2014) 1–18.
- [4] X. Li, Y. Hu, X. Gao, D. Tao, B. Ning, A multi-frame image super-resolution method, *Signal Process.* 90 (2010) 405–414.
- [5] L. Zhang, Q. Yuan, H. Shen, P. Li, Multiframe image super-resolution adapted with local spatial information, *J. Opt. Soc. Am. A* 28 (2011) 381–390.
- [6] X. Zeng, L. Yang, A robust multiframe super-resolution algorithm based on half-quadratic estimation with modified BTv regularization, *Digit. Signal Process.* 23 (2013) 98–109.
- [7] L. Pickup, S. Roberts, A. Zisserman, D. Capel, Multiframe super-resolution from a Bayesian perspective, *Super-Resolut. Imaging* (2010) 247.
- [8] B. Maiseli, O. Elisha, J. Mei, H. Gao, Edge Preservation Image Enlargement and Enhancement Method Based on the Adaptive Perona–Malik Non-Linear Diffusion Model, 2014.
- [9] R. Hardie, A fast image super-resolution algorithm using an adaptive Wiener filter, *IEEE Trans. Image Process.* 16 (2007) 2953–2964.
- [10] R.C. Hardie, K.J. Barnard, Fast super-resolution using an adaptive Wiener filter with robustness to local motion, *Opt. Express* 20 (2012) 21053–21073.
- [11] T.Q. Pham, L.J. Van Vliet, K. Schutte, Robust fusion of irregularly sampled data using adaptive normalized convolution, *EURASIP J. Adv. Signal Process.* 2006 (2006).
- [12] A. Marquina, S.J. Osher, Image super-resolution by TV-regularization and Bregman iteration, *J. Sci. Comput.* 37 (2008) 367–382.
- [13] Q. Yuan, L. Zhang, H. Shen, Multiframe super-resolution employing a spatially weighted total variation model, *IEEE Trans. Circuits Syst. Video Technol.* 22 (2012) 379–392.
- [14] B. Wu, E.A. Ogada, J. Sun, Z. Guo, A total variation model based on the strictly convex modification for image denoising, in: *Abstract and Applied Analysis*, vol. 2014, Hindawi Publishing Corporation, Nasr, 2014.
- [15] E.A. Ogada, Z. Guo, B. Wu, An alternative variational framework for image denoising, in: *Abstract and Applied Analysis*, vol. 2014, Hindawi Publishing Corporation, Nasr, 2014.
- [16] F. Knoll, K. Bredies, T. Pock, R. Stollberger, Second order total generalized variation (TGV) for MRI, *Magn. Reson. Med.* 65 (2011) 480–491.
- [17] Z. Guo, J. Sun, D. Zhang, B. Wu, Adaptive Perona–Malik model based on the variable exponent for image denoising, *IEEE Trans. Image Process.* 21 (2012) 958–967.
- [18] J. Weickert, A review of nonlinear diffusion filtering, in: *Scale-Space Theory in Computer Vision*, Springer, Berlin Heidelberg, 1997, pp. 1–28.
- [19] J. Weickert, Coherence-enhancing diffusion filtering, *Int. J. Comput. Vis.* 31 (1999) 111–127.
- [20] P. Perona, J. Malik, Scale-space and edge detection using anisotropic diffusion, *IEEE Trans. Pattern Anal. Mach. Intell.* 12 (1990) 629–639.
- [21] Y.-L. You, W. Xu, A. Tannenbaum, M. Kaveh, Behavioral analysis of anisotropic diffusion in image processing, *IEEE Trans. Image Process.* 5 (1996) 1539–1553.
- [22] M.J. Black, G. Sapiro, D.H. Marimont, D. Heeger, Robust anisotropic diffusion, *IEEE Trans. Image Process.* 7 (1998) 421–432.

- [23] S. Kichenassamy, The Perona–Malik paradox, *SIAM J. Appl. Math.* 57 (1997) 1328–1342.
- [24] P. Charbonnier, L. Blanc-Feraud, G. Aubert, M. Barlaud, Two deterministic half-quadratic regularization algorithms for computed imaging, in: *Proceedings of the IEEE International Conference on Image Processing, ICIP-94*, vol. 2, IEEE, New York, 1994, pp. 168–172.
- [25] J. Weickert, B.T.H. Romeny, M.A. Viergever, Efficient and reliable schemes for nonlinear diffusion filtering, *IEEE Trans. Image Process.* 7 (1998) 398–410.
- [26] H. Bouzari, An improved regularization method for artifact rejection in image super-resolution, *Signal Image Video Process.* 6 (2012) 125–140.
- [27] H. Kim, K.-S. Hong, Variational approaches to super-resolution with contrast enhancement and anisotropic diffusion, *J. Electron. Imaging* 12 (2003) 244–251.
- [28] H. Kim, J.-H. Jang, K.-S. Hong, Edge-enhancing super-resolution using anisotropic diffusion, in: *Proceedings of the International Conference on Image Processing*, vol. 3, IEEE, New York, 2001, pp. 130–133.
- [29] J. Weickert, *Anisotropic Diffusion in Image Processing*, vol. 1, Teubner Stuttgart, Valladolid, 1998.
- [30] A.N. Tikhonov, V.Y. Arsenin, *Solutions of Ill-Posed Problems*, 1977.
- [31] L.I. Rudin, S. Osher, E. Fatemi, Nonlinear total variation based noise removal algorithms, *Physica D: Nonlinear Phenom.* 60 (1992) 259–268.
- [32] S. Ganan, D. McClure, *Bayesian Image Analysis: An Application to Single Photon Emission Tomography*, American Statistical Association, Boston, Massachusetts, pp. 12–18.
- [33] S. Farsiu, M.D. Robinson, M. Elad, P. Milanfar, Fast and robust multiframe super resolution, *IEEE Trans. Image Process.* 13 (2004) 1327–1344.
- [34] B. Maiseli, C. Wu, J. Mei, Q. Liu, H. Gao, A robust super-resolution method with improved high-frequency components estimation and aliasing correction capabilities, *J. Frankl. Inst.* 351 (2014) 513–527.
- [35] R.M. Bahy, G.I. Salama, T.A. Mahmoud, Adaptive regularization-based super resolution reconstruction technique for multi-focus low-resolution images, *Signal Process.* 103 (2014) 155–167.
- [36] S. Villena, M. Vega, S.D. Babacan, R. Molina, A.K. Katsaggelos, Bayesian combination of sparse and non-sparse priors in image super resolution, *Digit. Signal Process.* 23 (2013) 530–541.
- [37] D. Kim, H. Byun, Regularization based super-resolution image processing algorithm using edge-adaptive non-local means filter, in: *Proceedings of the 7th International Conference on Ubiquitous Information Management and Communication*, ACM, New York, p. 78.
- [38] N. Sochen, R. Kimmel, R. Malladi, A general framework for low level vision, *IEEE Trans. Image Process.* 7 (1998) 310–318.
- [39] G. Gilboa, *Super-Resolution Algorithms Based on Inverse Diffusion-Type Processes* (Ph.D. thesis), Technion–Israel Institute of Technology, 2004.
- [40] K. Höllig, Existence of infinitely many solutions for a forward backward heat equation, *Trans. Am. Math. Soc.* 278 (1983) 299–316.
- [41] I.J. Day, On the inversion of diffusion NMR data: Tikhonov regularization and optimal choice of the regularization parameter, *J. Magn. Reson.* 211 (2011) 178–185.
- [42] R. Courant, K. Friedrichs, H. Lewy, On the partial difference equations of mathematical physics, *IBM J. Res. Dev.* 11 (1967) 215–234.
- [43] Z. Wang, A.C. Bovik, Mean squared error: love it or leave it? A new look at signal fidelity measures, *IEEE Signal Process. Mag.* 26 (2009) 98–117.
- [44] Z. Wang, A.C. Bovik, H.R. Sheikh, E.P. Simoncelli, Image quality assessment: from error visibility to structural similarity, *IEEE Trans. Image Process.* 13 (2004) 600–612.
- [45] H. Scharf, J. Weickert, An anisotropic diffusion algorithm with optimized rotation invariance, in: *Mustererkennung 2000*, Springer, Berlin Heidelberg, 2000, pp. 460–467.
- [46] J. Weickert, H. Scharf, A scheme for coherence-enhancing diffusion filtering with optimized rotation invariance, *J. Vis. Commun. Image Represent.* 13 (2002) 103–118.
- [47] J. Weickert, *Coherence-enhancing shock filters*, in: *Pattern Recognition*, Springer, Berlin Heidelberg, 2003, pp. 1–8.
- [48] D. Martin, C. Fowlkes, D. Tal, J. Malik, A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics, in: *Proceedings of the Eighth IEEE International Conference on Computer Vision, ICCV 2001*, IEEE, New York, vol. 2, 2001, pp. 416–423.
- [49] P. Vandewalle, S. Sü, M. Vetterli, et al., A frequency domain approach to registration of aliased images with application to super-resolution, *EURASIP J. Adv. Signal Process.* 2006 (2006).
- [50] M. Irani, S. Peleg, Improving resolution by image registration, *CVGIP: Graph. Models Image Process.* 53 (1991) 231–239.
- [51] A. Zomet, A. Rav-Acha, S. Peleg, Robust super-resolution, in: *Proceedings of the 2001 IEEE Computer Society Conference on Computer Vision and Pattern Recognition, CVPR 2001*, vol. 1, IEEE, New York, 2001, p. I-645.
- [52] S. Panda, M. Prasad, G. Jena, Pocs Based Super-Resolution Image Reconstruction Using an Adaptive Regularization Parameter, [arxiv:1112.1484](https://arxiv.org/abs/1112.1484), 2011.
- [53] B.J. Maiseli, Q. Liu, O.A. Elisha, H. Gao, Adaptive Charbonnier super-resolution method with robust edge preservation capabilities, *J. Electron. Imaging* 22 (2013) 043027.